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Incorporating imaginary faults in graphical stress methods

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ABSTRACT

The concept of an imaginary fault is introduced in this communication as an auxiliary constraint in the graphical determination of principal stress directions. A new graphic method is developed for this purpose by applying the right dihedra method to both real faults and their imaginary faults. Although similar to each other, this method has advantages in simplicity and convenience over the right trihedra method that is essentially an extension of the right dihedra method.

1. Introduction

Inverting palaeostress in faulted rocks from measured fault/slip data was made possible by the pioneering works of Bott (1959), Carey and Brunier (1974), and others. Such work has been continued with vigour (see Ramsay and Lisle, 2000), because it can reveal detailed information on palaeostress, namely, principal directions and stress ratio, that is crucial to understand the past brittle deformation in the upper crust, from newly formed or activated or reactivated faults.

Auxiliary planes have been introduced in simple graphical determination of principal stress directions using the right dihedra method (Angelier and Mechler, 1977; McKenzie, 1969; Pegoraro, 1972). In the method, for each individual fault, the maximum and minimum stresses (σ_1 and σ_3) are confined to the dihedra or the fields bounded by the fault plane and an imaginary plane being normal to the slip line on the fault surface (Fig. 1a). The confidence fields for the two principal stresses are reduced in size when more faults are taken into consideration. More significantly, by use of the right trihedra method (Lisle, 1987; Ramsay and Lisle, 2000), these fields are further reduced in size through relocating the maximum and minimum principal stresses in the dihedra bounded by two auxiliary planes, one being normal to the slip line and another being perpendicular to the fault plane through the slip line, for each individual fault (see Fig. 1b).

The contribution of this note is to introduce the concept of an imaginary fault that helps simplify the connection between the right dihedra and right trihedra methods, and from which a new graphic method is developed to determine the principal stress directions. This proposed method is similar to the right trihedra method, but has some advantages.

2. Concept of imaginary fault

Let us consider a fault plane having a normal direction n and a slip direction s. Fundamental to inversion of stress from fault/slip data is the assumption of null traction exerted on the fault plane perpendicular to the slip line (e.g., Angelier, 1979):

$$n \cdot \sigma \cdot t = 0 \quad (1)$$

where σ is the controlling stress, and t is the unit vector on the fault plane perpendicular to the slip line.

Because the symmetry of the stress tensor enables the permutation between t and n, Eq. (1) can be written in another format as

$$t \cdot \sigma \cdot n = 0 \quad (2)$$

In the light of similarity between Eqs. (1) and (2), we have an imaginary fault with a normal direction of t and a slip direction of s, in respect to the real fault, that also satisfies the above-mentioned

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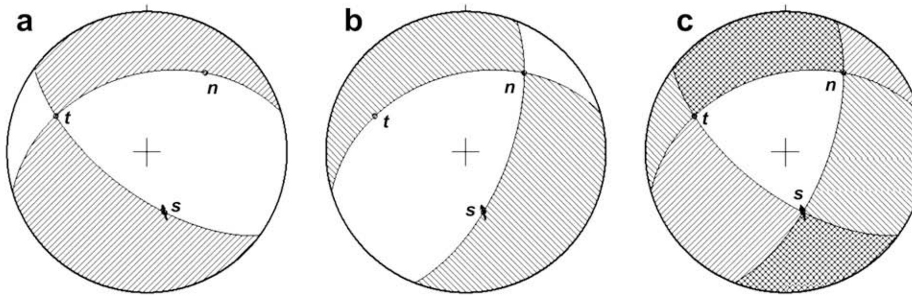

Fig. 1. For a given real fault with a normal (n) and a slip line (s), results on the stereogram from applying the right dihedra method (Angelier and Mechler, 1977) to the fault (a), its imaginary fault (b), and both (c). There is no difference in slip line or in slip sense between the real fault and its imaginary fault whose normal (t) is perpendicular to vectors s and n. The direction of arrows represents the relative movements of the hangingwalls. In (a) and (b), the maximum principal stress (σ_1) is located in the fields filled with both slashes and back slashes, while the minimum principal stress (σ_3) in the empty fields. Superimposing (a) and (b) reduces these confidence fields (c) half in size. Planes in (b) correspond to the auxiliary planes in the right trihedra method (Lisle, 1987; see the text for more explanation). Equal-area, lower hemispheric projection.

assumption. There are two options about the slip sense of the imaginary fault; with or against the slip vector of the real fault, and their determination is beyond either Eq. (1) or Eq. (2).

For the sake of simplicity, it is justifiable to transform through rotations the considered stress into an Andersonian orientation that has the maximum, intermediate and minimum principal stresses coincident with coordinate axes x, y, and z, respectively. That is to say, after transformation, for the prevailing stress that activates or reactivates the real fault,

$$\sigma_{ij} = 0 \quad (i \neq j; i,j = 1,2,3), \quad \text{and} \quad \sigma_{11} \geq \sigma_{22} \geq \sigma_{33}. \tag{3}$$

where σ_{ij} are stress elements.

Let us consider in the transformed state a real fault plane whose normal (n) has directional cosines n_1, n_2, and n_3 with respect to coordinate axes x, y, and z. The maximum shear stress (τ) on the fault plane, parallel to the slip direction as assumed in stress inversion (e.g., Angelier, 1979), is calculated according to the following expression:

$$\begin{aligned}\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix} &= \begin{bmatrix}\sigma_{11} & 0 & 0\\ 0 & \sigma_{22} & 0\\ 0 & 0 & \sigma_{33}\end{bmatrix}\begin{bmatrix}n_1\\ n_2\\ n_3\end{bmatrix} - \left(\begin{bmatrix}n_1 & n_2 & n_3\end{bmatrix}\right.\\ &\quad \left.\times \begin{bmatrix}\sigma_{11} & 0 & 0\\ 0 & \sigma_{22} & 0\\ 0 & 0 & \sigma_{33}\end{bmatrix}\begin{bmatrix}n_1\\ n_2\\ n_3\end{bmatrix}\right)\begin{bmatrix}n_1\\ n_2\\ n_3\end{bmatrix}\\ &= \begin{bmatrix}\left[(\sigma_{11}-\sigma_{22})n_2^2 + (\sigma_{11}-\sigma_{33})n_3^2\right]n_1\\ \left[(\sigma_{22}-\sigma_{11})n_1^2 + (\sigma_{22}-\sigma_{33})n_3^2\right]n_2\\ \left[(\sigma_{33}-\sigma_{11})n_1^2 + (\sigma_{33}-\sigma_{22})n_2^2\right]n_3\end{bmatrix}\end{aligned} \tag{4}$$

An imaginary fault that passes through the real slip line and normal to the real fault plane has directional cosines t_1, t_2, and t_3 proportional to (Jeager, 1962, p. 18):

$$\begin{bmatrix}t_1\\ t_2\\ t_3\end{bmatrix} = k\begin{bmatrix}(\sigma_{33}-\sigma_{22})n_2n_3\\ (\sigma_{11}-\sigma_{33})n_1n_3\\ (\sigma_{22}-\sigma_{11})n_1n_2\end{bmatrix} \tag{5}$$

where k is an auxiliary constant that keeps the vector at the left side in unit length. In practice, both t_3 and n_3 have the same sign, thus requiring

$$-kn_1n_2n_3 \geq 0 \tag{6}$$

Similarly, according to Eq. (5), the maximum shear stress (τ') on the imaginary fault plane is expressed as follows,

$$\begin{bmatrix}\tau'_1\\ \tau'_2\\ \tau'_3\end{bmatrix} = \begin{bmatrix}\left[(\sigma_{11}-\sigma_{22})n_2^2+(\sigma_{11}-\sigma_{33})n_3^2\right]n_1(-wkn_1n_2n_3)\\ \left[(\sigma_{22}-\sigma_{11})n_1^2+(\sigma_{22}-\sigma_{33})n_3^2\right]n_2(wkn_1n_2n_3)\\ \left[(\sigma_{33}-\sigma_{11})n_1^2+(\sigma_{33}-\sigma_{22})n_2^2\right]n_3(-wkn_1n_2n_3)\end{bmatrix} \tag{7}$$

where $w = (\sigma_{11}-\sigma_{22})(\sigma_{11}-\sigma_{33})(\sigma_{22}-\sigma_{33})k^2 \geq 0$. Because of Ineq. (6), there is no difference in sign between τ and τ' (compare Eqs. (4) and (7)), indicating that both the real and imaginary faults possess a common unit slip vector.

With respect to the real fault, the imaginary fault makes no contribution to solving Eq. (1) or Eq. (2) or both for stress, because the equations are strictly similar to each other. However, the imaginary fault is of practical value in determining stress graphically, as described in the following section.

3. Graphic determination

By use of the right dihedra method (Angelier and Mechler, 1977), both the real faults and their imaginary faults are used together to look for the confidence fields for the maximum or the minimum principal stresses (Fig. 1). In comparison with those fields (Fig. 1a) simply based upon the real faults, these fields (Fig. 1c) are substantially reduced, half in area for a real and an imaginary fault, for instance. This superimposition increases the efficiency of the method without the need of more fault measurements.

Fig. 2 shows an example of three artificial fault/slip data generated under a certain prescribed stress, to which the modified right dihedra method proposed above is applied. In this case, we are satisfied with the resulting fields (Fig. 2c) for the maximum and minimum principal stresses (Fig. 2d), because they have much smaller area than those counterparts in Fig. 2b.

4. Discussion

Interestingly, the two auxiliary planes introduced by Lisle (1987), one being normal to the slip line and another being perpendicular to the real fault plane through the slip line, correspond to the imaginary fault plane and a plane perpendicular to the imaginary fault plane through the slip line. It is thus easy to extend from the right dihedra method the right trihedra method developed by Lisle (1987), in the light of incorporating the imaginary fault(s) to the graphical determination of principal orientations. Theoretically speaking, our method proposed in this paper appears simpler than the right trihedra method, as was previously described. It is more convenient to use our proposed method to determine how the principal stresses, σ_1 and σ_3 are located within the dihedra confined by the auxiliary planes, because both the real fault and its imaginary fault possess the same unit slip vector.

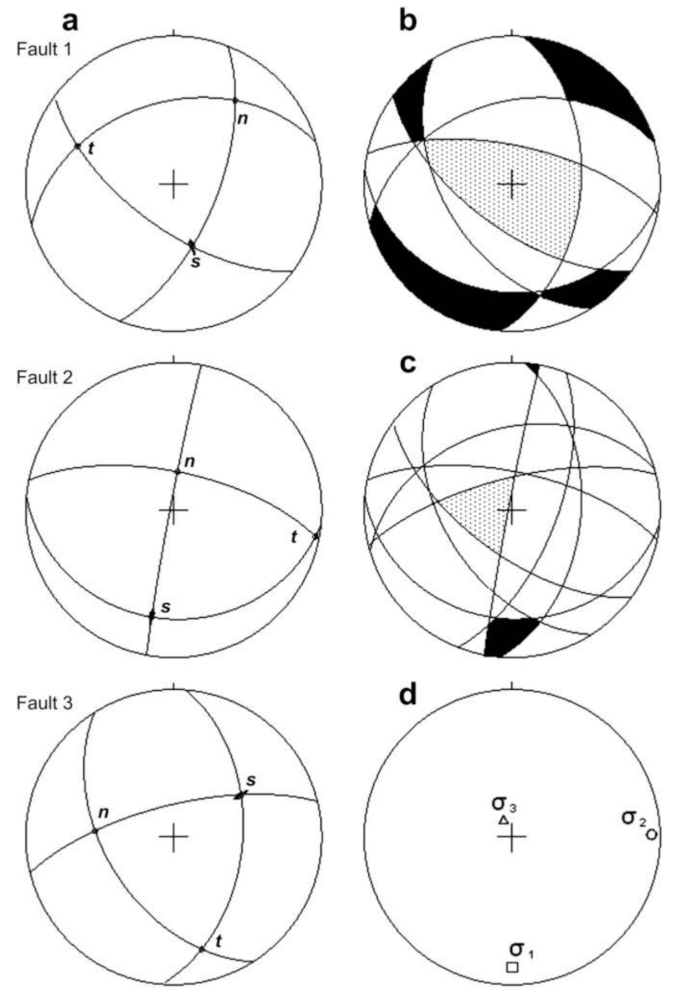

Fig. 2. For a set of three real fault/slip data (a) generated under a certain prescribed stress (d), results on the stereogram from applying the right dihedra method (a) and the method proposed in this paper (b) to them. In (b) and (c), the maximum principal stress (σ_1) is within the black field(s), while the minimum principal stress (σ_3) with the dotted field(s). Equal-area, lower hemispheric projection.

In the case that the faulted region underwent axial compression or axial extension, the imaginary fault planes of a single tectonic stage intersect or tend to intersect at σ_1 or σ_3 (Arthaud, 1969). Aleksandrowski (1985) applied this idea to a general case of triaxial stress, by identifying the most dense intersection point(s) on stereonet of these great circles as either of the two principal stress directions. His application remains unsound, because the imaginary faults play a simple role in confining the confidence fields for these principal directions.

5. Conclusions

For a given real fault, an imaginary fault whose fault plane goes through and the real slip direction and the normal to the real fault plane, is introduced in this paper. It has the same slip line and the same slip sense as the real fault. The concept of an imaginary fault is of value in simplifying the connection between the right dihedra and right trihedra methods: the latter is a simple extension of the former. A new graphic method is developed to determine principal stress directions by applying the right dihedra method to the real and imaginary faults together. This method is similar to the right trihedra method, but has advantages of simplicity and convenience.

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